

Section 2.4

Grade: «grade»
Subject: «subject»
Date: «date»

Lab: Zooming In

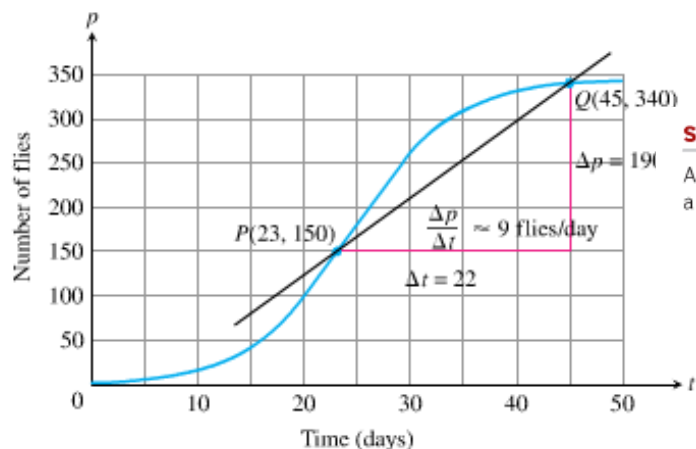
EXAMPLE 1 Finding Average Rate of Change

Find the average rate of change of $f(x) = x^3 - x$ over the interval $[1, 3]$.



$$\begin{array}{cc} P(1, f(1)) & Q(3, f(3)) \\ \downarrow & \\ (1, 0) & (3, 24) \end{array}$$

$$\frac{24 - 0}{3 - 1} = \frac{24}{2} = 12$$



Secant to a Curve

A line through two points on a curve is a **secant to the curve**.

Figure 2.27 Growth of a fruit fly population in a controlled experiment.

Source: *Elements of Mathematical Biology*. (Example 2)

EXAMPLE 2 Growing *Drosophila* in a Laboratory

Use the points $P(23, 150)$ and $Q(45, 340)$ in Figure 2.27 to compute the average rate of change and the slope of the secant line PQ .

$$\frac{340-150}{45-23} \approx 8.64 \text{ flies/day}$$

As suggested by Example 2, we can always think of an average rate of change as the slope of a secant line.

Secant-tangent of curve.gsp

Q	slope of line PQ

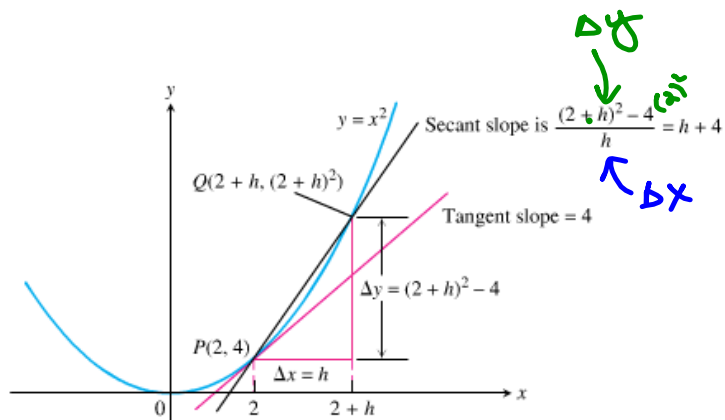
Tangent to a Curve

1. We start with what we can calculate, namely, the slope of a secant through P and a point Q nearby on the curve.
2. We find the limiting value of the secant slope (if it exists) as Q approaches P along the curve.
3. We define the *slope of the curve at P* to be this number and define the *tangent to the curve at P* to be the line through P with this slope.

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

EXAMPLE 3 Finding Slope and Tangent Line

Find the slope of the parabola $y = x^2$ at the point $P(2, 4)$. Write an equation for the tangent to the parabola at this point.



DEFINITION Slope of a Curve at a Point

The **slope of the curve** $y = f(x)$ at the point $P(a, f(a))$ is the number

$$m = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h},$$

provided the limit exists.

EXAMPLE 4 Exploring Slope and Tangent

Let $f(x) = 1/x$.

- (a) Find the slope of the curve at $x = a$.
- (b) Where does the slope equal $-1/4$?
- (c) What happens to the tangent to the curve at the point $(a, 1/a)$ for different values of a ?

The expression

$$\frac{f(a+h) - f(a)}{h}$$

is the **difference quotient** of f at a .

- 1 Find the slope at the point P(2,4) of
 $y = x^2$

Normal to a Curve

The **normal line** to a curve at a point is the line perpendicular to the tangent at that point.

EXAMPLE 5 Finding a Normal Line

Write an equation for the normal to the curve $f(x) = 4 - x^2$ at $x = 1$.



2 Answer?

$$\lim_{h \rightarrow 0} \frac{f(t + h) - f(t)}{h}.$$

EXAMPLE 6 Investigating Free Fall

Find the speed of the falling rock in Example 1, Section 2.1, at $t = 1$ sec.